DBZ-003-1162004

Seat No. ________

M. Sc. (Sem. II) Examination

July - 2022

Mathematics : CMT-2004

(Methods in Partial Differential Equation)

Faculty Code : 003
Subject Code : 1162004

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are five questions.
(2) All questions are compulsory.
(3) Each question carries 14 marks.

1 Answer any seven from the following : **14**

(a) Define (i) Complete solution (ii) Pffafian form with each of its example.

(b) Solve, $\left(\frac{\partial^3 z}{\partial z^3} - 3\frac{\partial^3 z}{\partial^2 x\partial y} + 2\frac{\partial^3 z}{\partial y^3}\right) = 0.$

(c) Find the differential equation of all the spheres whose centre lie on the z-axis.

(d) Find the complete integral of $p^2 = q + x$.

(e) Is the equation $6ydx - 7y^2dx = (4x^2)dy$ is exact ? Justify your answer.

(e) Is the equation $6ydx - 7y^2dx = \left(4x^2\right)dy$ is exact ? Justify your answer.

(f) Verify the equation $z^2 + \mu = 2(1+\gamma^{-1})(x+\gamma y)$ is the complete integral of $z = \frac{1}{p} + \frac{1}{q}$.

(g) Find the particular integral $(D^2 - D')z = 2y - x^2$.

(h) Write down two sets of parametric equations for sphere.

(i) Find the direction cosines of the normal to the surface $x+y+z=10$ at point $p(1,1,1)$.

(j) Determine the envelope for the equation $(x-a)^2+(y-b)^2+z^2=1$.

2 Answer any two of the following : **14**

(1) State and prove, the necessary and sufficient condition under which the Pfaffian differential equation (for three variables) is integrable.

(2) If $(\alpha D+\beta D'+\gamma)^2$ with $\alpha \neq 0$ is a factor of $F(D, D')$, then a solution of the equation $F(D, D')$ is,

$$z = e^{-\frac{\gamma}{\alpha}x}(\varnothing_1(\beta x-\alpha y)+y\varnothing_2(\beta x-\alpha y))$$

(3) Solve, $(2D^2-5DD'+2D'^2)z = \sin\ (2x+y)$

3 Answer the following : **14**

(1) (i) Solve, the partial differential equation :

$$x(x^2+3y^2)p-y(y^2+3x^2)q-2z(y^2-x^2)=0.$$

(ii) Find the solution of $y^2p-xyq=x(z-2y)$.

(2) Solve by Nattani's method

$$yzdx-(x^2y-zx)dy+(x^2z-xy)dz=0.$$

OR

3 Answer the following : **14**

(1) Solve by Charpits method $2(z+xp+yq)=yp^2$

(2) Write down, how to solve a partial differential equation $f(x,u_1,u_3)=g(y,u_2,u_3)$ using Jacobi's method and illustrate the method by finding a complete integral of $2x^2u_1^2yu_3=x^2u_2+2yu_1^2$?

4 Answer the following : **5+5+4 = 14**

(1) Solve by Jacobi's method $(p^2+q^2)y=qz$.

(2) Prove that, the relation $F(U, V)=0$ where U and V are the functions of x, y and z obtain the partial differential equation of the type $Pp+Qq=R$.

(3) Solve, $4r+12s+9t=e^{3x-2y}$.

5 Answer any two of the following : **14**

(1) Classify the equation and convert it in canonical form $y^2r+4x^2t=xy(x \neq 0 \neq y)$.

(2) (i) Solve the equation

$$\frac{dx}{xy+y^2+2z}=\frac{dy}{x^2+xy-2z}=\frac{dz}{z(x+y)}$$

(ii) Form the P.D.E. from $z = xf(y) + yg(x)$

(3) Find the integral surface of the given partial differential equation $(2xy-1)p + (z-2x^2)q = 2(x-yz)$, which passes through the lines $y=0$ and $x=1$

(4) Find the orthogonal trajectories on the surface $(x+y)z=1$ of its curve of intersection with the system of planes $x-y+z = k$ where k is a parameter.